Market Model Regression

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BUSI 448: Investments

Where are we?

Last time:

- Input sensitivity

Today:

- Market Model Regressions
- Alphas and Betas
- Estimating the Covariance Matrix
- Estimation Error

Single Benchmark Models

Benchmark Regression

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i \left(r_{b,t} - r_{f,t} \right) + \varepsilon_{i,t}$$

- Regress stock excess returns on benchmark excess returns
- $\beta_i = \frac{\text{cov}(r_i - r_f, r_b - r_f)}{\text{var}(r_b - r_f)}$
- Most common benchmark is a market return
  - CRSP value-weighted market, S&P 500
  - I'll refer to this as the **market model**

Understanding the Market Model Regression

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i (r_{m,t} - r_{f,t}) + \varepsilon_{i,t}$$

- Meaning of α?
- Meaning of β?
- Meaning of ε?

Visualization

Meaning of β

Beta answers the question:

if the benchmark is up 1%, how much do we expect the asset to be up, all else equal?

- **If β=2, we expect the asset to be up 2%**
- **If β=0.5, we expect the asset to be up 0.5%**

Meaning of α

Alpha answers the question:

if I were holding the market, could I have improved mean-variance efficiency by investing something in the asset?

- **The answer is "yes" if and only if $\alpha > 0$**
- **If $\alpha < 0$, mean-variance efficiency could have been improved by shorting the asset.**

Visualization

A warning

- Alphas with respect to a benchmark regression are **backward-looking**.
- We should only use them for forming portfolios if we believe that the alpha will persist!

Estimating Covariances

Number of Parameters

How many parameters do we need to estimate for an N asset covariance matrix?

$$\begin{bmatrix} \text{var}[r_1] & \text{cov}[r_1, r_2] & \dots & \text{cov}[r_1, r_N] \\ \text{cov}[r_2, r_1] & \text{var}[r_2] & \dots & \text{cov}[r_2, r_N] \\ \vdots & \vdots & \ddots & \vdots \\ \text{cov}[r_N, r_1] & \text{cov}[r_N, r_2] & \dots & \text{var}[r_N] \end{bmatrix}$$

How many variance terms?

$$N$$

How many distinct covariance terms?

$$\frac{N^2 - N}{2}$$

Curse of Dimensionality

N(Assets)	N(Parameters)
5	15
10	55
25	325
50	1,275
100	5,050

- A great deal of estimation risk with 5,000 parameters to estimate!

Market Model-Implied Covariances

Under the market model, what is the covariance of two assets i and j, $\text{cov}(r_i, r_j)$?

$$\text{cov}(\alpha_i + \beta_i(r_m - r_f) + \varepsilon_i, \alpha_j + \beta_j(r_m - r_f) + \varepsilon_j)$$

- The alphas are constant, so we can ignore them.
- If we are willing to assume that ε_i is uncorrelated with ε_j, the covariance reduces to:

$$\beta_i \beta_j \text{var}(r_m - r_f)$$

Market Model-Implied Variances

For variance terms, we definitely should not ignore the residual variance:

$$\text{var}(r_i) = \beta_i^2 \text{var}(r_m) + \text{var}(\varepsilon_i)$$

Alternatively, we can just estimate the stock-specific variance directly.

Reduced parameter dimensionality

- Pairwise ρ: $\frac{N^2-N}{2}$ correlations, N variances
- Market Model: N betas, N variances, 1 mkt variance

N(Assets)	Pairwise ρ N(Parameters)	Market Model N(Parameters)
5	15	11
10	55	21
50	1,275	101
100	5,050	201

Industry Portfolios

- **Notebook #1**: Estimate betas for industry portfolios and calculate market model-implied covariance matrix
- **Notebook #2**: Backtest performance of using the market model-implied covariance matrix for industry portfolios

Persistence of β (and α)

Estimation error

- Alpha and beta are estimates, so will be subject to the usual concerns about estimation error.

Shrinking betas

- On average, what value should beta have?
- A simple way to deal with estimation error is to shrink betas towards 1.

$$\beta_{\text{adjusted}} = 0.67 \cdot \beta_{\text{adjusted}} + 0.33 \cdot 1$$

- Many fancier alternatives exist.

Let’s return to notebook #1 and consider how well shrinking betas performs for our industry portfolios.

For next time: “Capital Asset Pricing Model”