CAPM

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BUSI 448: Investments

Where are we?

Last time:

- Market Model Regressions
- Alphas and Betas
- Estimating Covariance Matrix

Today:

- CAPM and the Market Model Regression
- CAPM: Theory
- CAPM: Practice

CAPM and the Market Model Regression

What is the CAPM?

- The Capital Asset Pricing Model (CAPM) is a theory from the 1960s. Its discoverer won the Nobel prize in economics.
- The intuition is:
  - Market risk is the biggest risk that a diversified investor faces.
  - The risk of each asset should be measured in terms of how much it contributes to market risk.
  - The risk premium of each asset should depend (linearly) on this measure of risk.

Capital Asset Pricing Model

$$E[r_i - r_f] = \beta_i \cdot E[r_m - r_f]$$

- $E[r_m - r_f]$ is the market risk premium
- $\beta_i = \frac{\text{cov}(r_i, r_m)}{\text{var}(r_m)}$

Empirically, we estimate a **market model regression**:

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i (r_{m,t} - r_{f,t}) + \varepsilon_{i,t}$$

- What differs between the top and bottom equations?

Theory: CAPM

CAPM Assumptions

- Investors have **identical** beliefs about the **same universe** of asset returns
- Investors have **mean-variance** preferences
- **Single period** investment horizon

CAPM Assumptions

- **Frictionless** borrowing and lending
  - borrowing rate = savings rate
- **Frictionless** trading
  - no transactions costs & no taxation & shorting allowed
- **Perfect competition**: investors are price-takers

Equilibrium

- All investors view the market portfolio as the tangency portfolio.
- The capital allocation line with respect to the market portfolio is called the **capital market line**
- Investors save or borrow at the risk-free rate to locate on the CML according to their risk aversion.
- Prices will adjust so that the marginal benefit of an asset (its risk premium) is proportional to its marginal contribution to the risk of the *market portfolio*.

Deriving the CAPM (1/3)

Recall that the tangency portfolio in a frictionless setting satisfies:

$$\sum_{i=1}^{N} \text{cov}[r_1, r_i] w_i = \delta(E[r_1] - r_f)$$
$$\sum_{i=1}^{N} \text{cov}[r_2, r_i] w_i = \delta(E[r_2] - r_f)$$
$$\vdots$$
$$\sum_{i=1}^{N} \text{cov}[r_N, r_i] w_i = \delta(E[r_N] - r_f)$$

where δ is a constant (it is a Lagrange multiplier from the optimization problem)

- The LHS terms are the contributions of each asset to overall portfolio risk.
- The RHS terms are proportional to each asset's risk premium.

Deriving the CAPM (2/3)

- Previously: we solved the system for weights
- CAPM: solve for expected returns using market weights

For asset j:

$$\sum_{i=1}^{N} \text{cov}[r_j, r_i] w_i = \delta(E[r_j] - r_f)$$

Rearrange and use the fact that $r_m = \sum_i w_i r_i$ to get:

$$E[r_j - r_f] = \delta^{-1} \text{cov}[r_j, r_m]$$

Deriving the CAPM (3/3)

Using the definition of beta:

$$E[r_j - r_f] = \delta^{-1} \beta_j \text{var}[r_m] \, .$$

Now aggregate this at market weights:

$$\sum_j w_j \cdot E[r_j - r_f] = \delta^{-1} \text{var}[r_m] \sum_j w_j \cdot \beta_j$$

This implies $\delta = \text{var}[r_m]/E[r_m - r_f]$, so we arrive at the CAPM formula:

$$E[r_j - r_f] = \beta_j E[r_m - r_f].$$

Intuition of the equilibium

- The marginal benefit of an asset (its risk premium) is proportional to its marginal contribution to the risk of the *market portfolio*
- The marginal contribution to risk is measured by beta.

What if this weren't the case?

- If an asset's reward to risk contribution ratio is higher than ratios for other assets, what would you do?
  - Hold the asset at a greater weight, reducing weights in others.
  - But purchasing would push price up and return down until all investments had the same reward-to-risk-contribution ratio.

Practice: CAPM

CAPM and Corporate Finance

- The CAPM is widely used to estimate expected returns to compute discount factors for corporate investment projects.
  - The return shareholders expect is $r_f + \beta_i \cdot E[r_m - r_f]$.
  - This is the required return on equity capital for corporate projects.
  - aka cost of equity capital

Website

CAPM and Investments

- The CAPM is somewhat less useful in an investments context.
  - What are the inputs for r_f and $E[r_m - r_f]$?
  - Estimating inputs can be too noisy
  - Doesn't describe the cross-section of equity returns well

Estimating the market risk premium

- Empirically, this is challenging.
- An additional complication: the MRP is likely time-varying.

Historical average market risk premium

- One option is to use the **realized** average:

$$\frac{1}{T}\sum_t (r_{m,t} - r_{f,t})$$

  as an estimate of the **expected** market risk premium

- Sample means are noisy estimates of population means
  - Need a large T sample
  - Precision of estimate doesn't improve with sampling data more frequently.

Precision of historical average

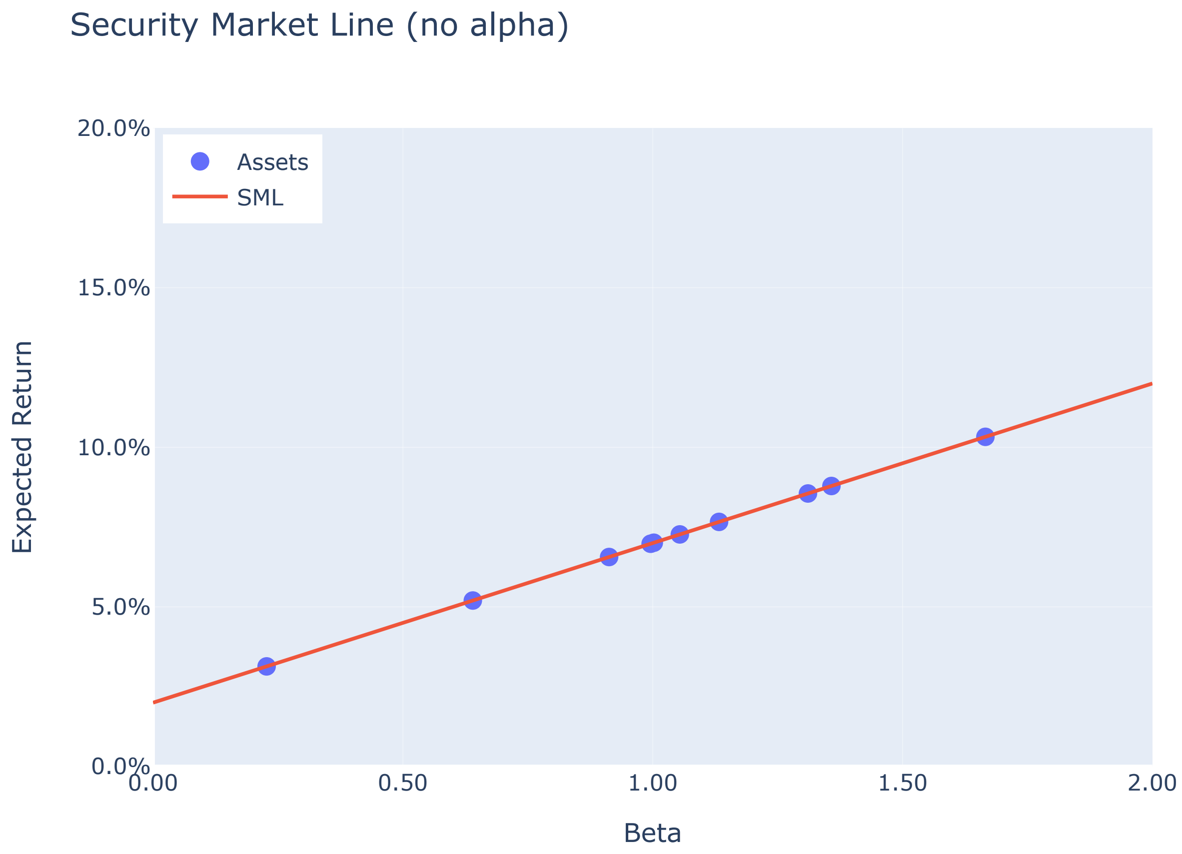
- Standard error = SD/$\sqrt{T}$
  - Annual SD of market return of 20%:

Years of Data	Standard Error of Estimates
5	8.94%
10	6.32%
25	4.00%
50	2.83%
100	2.00%

Visualization

Security market line

The **security market line** is the visual representation of the CAPM and the cross-section of expected returns



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The CAPM and cross-sectional data

- The CAPM doesn't fit realized returns in the cross-section of stocks very well.
- Theoretically, the slope of the SML should be:
  - $E[r_m - r_f]$
- Empirically, the slope is much flatter than the realized market risk premium.

Industry returns example

- A simple example is industry returns.
- Average returns are mostly unrelated to betas.

Website example

In-class notebook version

Let’s look at what this webpage is doing.

For next time: Predictability in the Cross-Section of Returns