Fixed Income: Duration

Kevin Crotty

BUSI 448: Investments

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Where are we?

Last time:

- Equity asset pricing models
- Multifactor models

Today:

- Interest rate risk
- Duration

Fixed Income Topics

- Interest rate risk
  - Duration
  - Convexity
- Credit risk
  - Leverage
  - Ratings
  - Credit default swaps
- Reinvestment risk

Bond pricing refresher

$$P = \frac{CF_1}{(1+y/m)} + \frac{CF_2}{(1+y/m)^2} + \ldots + \frac{CF_T}{(1+y/m)^T}$$

- m: number of payments per year
- y/m: per period yield (i.e., the discount rate)

For bonds, the cash flows are usually fixed coupon payments, so this reduces to:

$$P = \frac{C}{(1+y/m)} + \frac{C}{(1+y/m)^2} + \ldots + \frac{C + FACE}{(1+y/m)^T}$$

where C is the coupon payment of the bond.

Interest Rate Risk

Duration defined

$$P = \frac{C}{(1+DR)} + \frac{C}{(1+DR)^2} + \frac{C}{(1+DR)^3} + \ldots + \frac{C + FACE}{(1+DR)^T}$$

We can rewrite this as:

$$P = PV(CF_{t_1}) + PV(CF_{t_2}) + PV(CF_{t_3}) + \ldots + PV(CF_{t_T})$$

where t_1 is the time of the first cash-flow in years.

Now divide both sides by P:

$$1 = \frac{PV(CF_{t_1})}{P} + \frac{PV(CF_{t_2})}{P} + \frac{PV(CF_{t_3})}{P} + \ldots + \frac{PV(CF_{t_T})}{P}$$

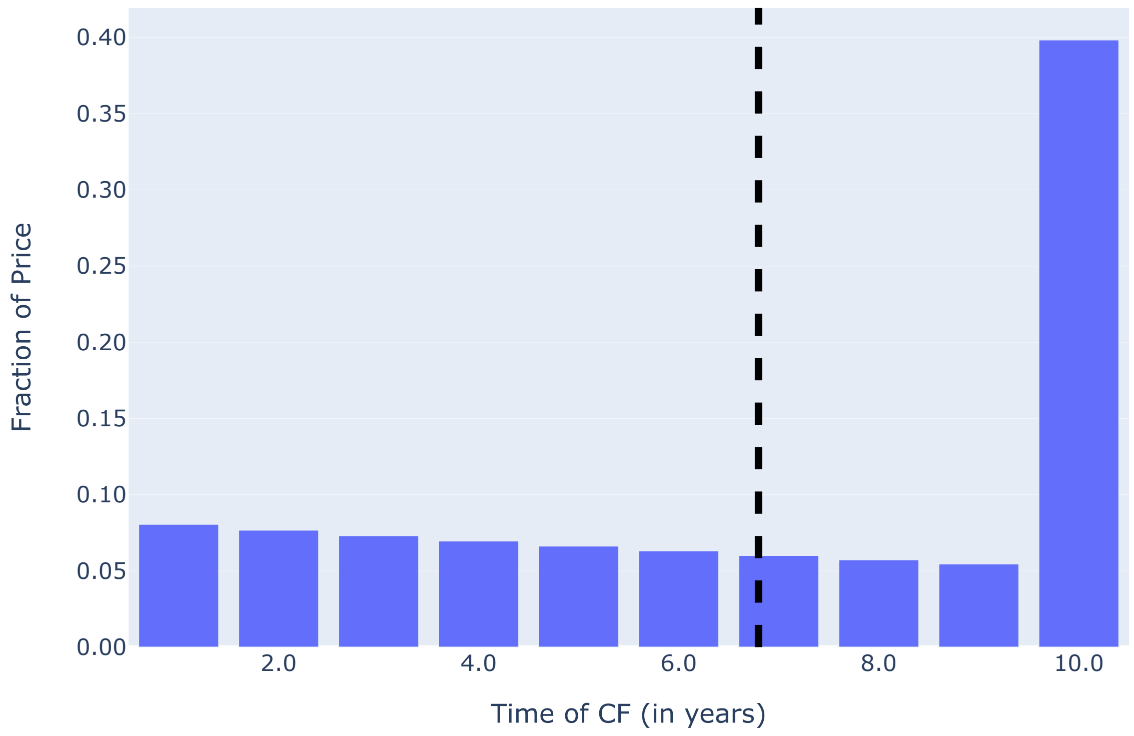
Each term on the RHS is a weight!

Duration defined

- Duration is a weighted-average time to cash flows.
- The weights are the fraction of the total PV (the price) that is due to the cash flows at each time.

$$\text{duration} = \left[\frac{PV(CF_{t_1})}{P}\right] \cdot t_1 + \left[\frac{PV(CF_{t_2})}{P}\right] \cdot t_2 + \ldots + \left[\frac{PV(CF_{t_T})}{P}\right] \cdot t_T$$

Duration visualized



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What happens to duration as:

- Maturity increases?
- Coupon rate increases?

Duration is related to interest rate risk!

For a change in yield y of Δy, the percent change in price is:

$$\frac{\Delta P}{P} \approx - \left[\frac{\text{duration}}{1 + DR}\right] \cdot \Delta y.$$

The term in brackets is **modified duration**.

Alternatively, we can work in prices rather than returns:

$$P_{\text{new}} \approx P_0 - P_0 \cdot \left[\frac{\text{duration}}{1 + DR}\right] \cdot \Delta y$$

Let's go to today's notebook and calculate duration

Duration and the bond pricing function

How good is this approximation?

Consider two bonds with

- Same YTM of 10%
- Same coupon rate of 5%
- Different maturities of 5 and 10 years

Let's look at how well the duration approximation works for different yield change magnitudes.

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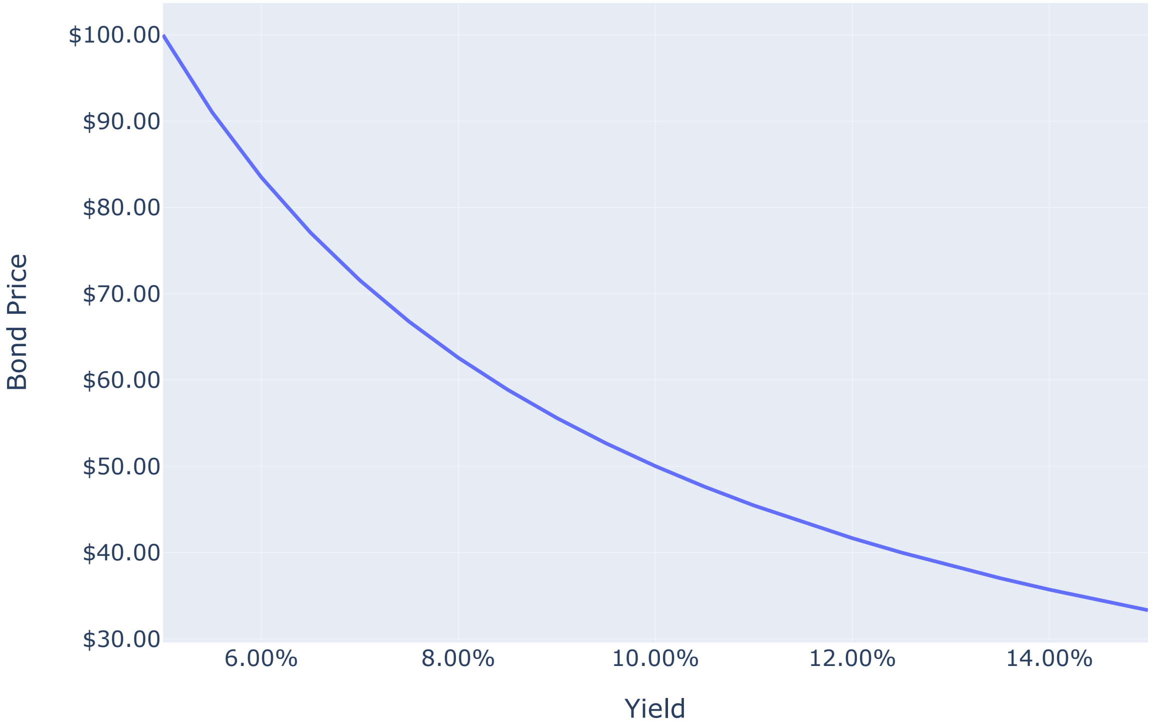
Drawbacks of duration

- Duration is a linear approximation.
- It can be improved using curvature of pricing function (convexity).
- Also, price risk is not the only risk associated with rate changes.
  - reinvestment risk!

Duration and reinvestment risk

Consider a rate decline from 10% to 9%



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Reinvestment risk

The risk that interest payments cannot be reinvested at the same rate.

- If rates fall
  - bond prices rise
  - but the value of reinvested coupons falls.

When investment horizon matches duration, reinvestment risk and interest rate risk cancel out!

An example

- Suppose you need to pay out $X at year 5 (think of a pension company).
- What is your investment strategy, using bonds, that ensures that you can meet your obligation?
- Best bet is to buy a zero-coupon bond maturing in 5 years
- If unavailable, buy a bond with duration of 5 years

Duration tells us three very useful things

- Effective maturity of a bond
- Interest rate risk (sensitivity of bond prices to rate changes)
- Horizon at which interest rate risk and reinvestment risk cancel out

For next time: Convexity